Who wants to Live Forever? Exploring the Frontiers of Human Life Span using Extreme Value Theory

Jorge Miguel Bravo[2]

Universidade Nova de Lisboa (NOVA IMS) & MagIC & CEFAGE-EU & Université Paris-Dauphine

Edviges Coelho

Statistics Portugal & Universidade Lusófona (ECEO-UHLT)

Extended Abstract

Increasing life expectancy at all ages in the developed world is one of the success stories of the last century. Improvements in survival are pushing new limits: today more than half of all males and two thirds of all females born in Western countries may reach their 80th birthday. The proportion of centenarians increased about ten times over the last thirty years, and more and more people celebrate their 100th birthday (Robine and Vaupel, 2001). These mortality improvements are a clear evidence of how far society and science have come in improving general living conditions, promoting healthier lifestyles, offering better medical and healthcare services that helped prolong our lives. As a result, the demographic structure of the population has changed significantly, with an increasing proportion to the overall mortality improvement in developed countries arising from a faster than expected decrease in mortality rates at advanced ages. Developments in the treatment of heart diseases, greater awareness of the dangers of smoking are just some of the reasons behind this trend that is reflected in the rapidly increasing number of centenarians in the industrialized world (Vaupel, 2010).

In recent decades, most OECD countries responded to continuous growth in life expectancy with pension reforms in which a common feature is to create an automatic link between future pensions and changes in life expectancy. The link between life expectancy and pension benefits has been accomplished in at least six different ways (Ayuso, Bravo & Holzmann, 2019): (i) By introducing an automatic link between life expectancy and pension benefits, for example through demographic

---

[2] Corresponding author: Universidade Nova de Lisboa NOVA IMS, Campus de Campolide, 1070-312 Lisboa I Portugal; Email: jbravo@novaims.unl.pt.

sustainability factors (e.g., Finland, Portugal, Spain); (ii) By linking the normal retirement age to life expectancy (so far 10 countries including Denmark, Italy, the Netherlands, and Portugal); (iii) By introducing FDC plans as a (often partial) replacement for unreformed NDB pensions (e.g., Mexico, Poland, Sweden); (iv) By connecting years of contributions needed for a full pension to life expectancy (e.g., France); (v) By substituting traditional NDB public schemes with NDC schemes that replicate some of the features of FDC plans, namely the way in which pension (annuity) benefits are computed (e.g., Sweden, Poland, Latvia, Italy, Norway); (vi) By linking penalties (bonuses) for early (late) retirement to years of contributions and normal retirement age (e.g., Portugal). This augmented the importance of using an appropriate measure of life expectancy estimate for pension policy, life insurance pricing and reserving, population forecasts and health care planning. The issue of correct estimation of future life expectancy is further complicated by the increasing recognition that longevity improvements are not homogenous across socioeconomic groups, creating non-intended tax/subsidy mechanisms that pervert redistribution in pension and other longevity risk pooling schemes and have consequences in the way longevity risk is shared between generations (Ayuso, Bravo, and Holzmann 2017a,b).

Although the evolution of mortality improvement is a slow but persistent process, influenced by socioeconomic, biological, environmental and behavioural developments, one of the most challenging tasks in longevity risk modeling is the issue of rare longevity events. The prediction of the tail of a survival distribution requires typically a special treatment due to the lack of high quality old-age mortality data. Specifically, the number of deaths and exposures-to-risk at advanced ages is often quite small, leading to significant sampling errors and highly volatile crude death rates. It is also well known that ages at death suffer from age heaping problems and are often misreported. To validate today's supercentenarians age at death a huge effort is required, and it is often impossible to confirm or dispute the accuracy of data in the absence of documentary evidence.

For demographers, actuaries and social planners, it has long been crucial to have a reliable model of old-age mortality for population forecasting, for pricing and reserve calculations and for capital market risk management solutions, particularly in products whose cash flows are contingent on survival or based a high-excess (longevity) layer (e.g., ALDA annuities). Life tables are the most popular instrument used to represent the underlying distribution of future lifetime variable and their construction relies on reliable mortality data. In the past, in the absence of appropriate mortality data at advanced ages actuaries very often neglected the importance of extreme longevity risk and arbitrarily adopted an ad-hoc procedure to close life tables by selecting an ultimate age, say 120, and setting the death probability at that age equal to 1, without any changes to adjacent mortality rates. This creates a discontinuity at the ultimate age compared to contiguous ages. Given the nature of mortality dynamics at these ages, the increasing importance of life expectancy estimates for public and private pension policy and (re)insurance, it is inaccurate to close life tables this way. The reason is that although the procedure may have a negligible impact on the expected value of future payments, the financial effect of underestimating the life table limiting age can be substantial, particularly in terms of risk measures such as VaR or Expected-Shortfall since these quantities heavily rely on the tail of the population survival distribution.

Various methodologies have been proposed for estimating mortality rates at oldest ages and for closing life tables within the insurance industry (see, for instance, Thatcher et al. 1998; Boleslawski and Tabeau, 2001; Buettner, 2002; Pitacco, 2004; Bravo et al., 2007). Ad-hoc methods include: (i) the forced method described above; (ii) a blended method consisting in selecting an ultimate age and blend the death probabilities from some earlier age to converge smoothly to 1.0 at the ultimate age; (iii) a pattern extrapolation method that simply consists in letting the pattern of mortality continue until the mortality quotient approaches or hits 1 and in setting that as the life table ultimate age; (iv) methods that involve selecting an ultimate age but end the table at whatever rate is produced by the extrapolation procedure at that age given that the ultimate death probability is less than 1.0. Other methods (e.g., method of extinct generations and the survivor ratio method) generate population numbers from death registrations, which for the purpose of estimating the number of very old people are considered more reliable than population estimates derived from censuses.

Alternative methodologies include fitting mortality curves over a certain age range, for which crude mortality rates may be calculated directly from data, followed by extrapolation. For example, the Coale-Kisker method (Coale and Guo, 1989; Coale and Kisker, 1990) assumes that the exponential rate of mortality increase at very old ages is not constant, as stipulated by the classical Gompertz (1825) and Heligman-Pollard (1980) models but declines linearly with age. Himes et al. (1994) presented a standard life table model for adult ages combined with a Brass-type logit relational model for mortality at the oldest-old ages. Denuit and Goderniaux (2005) perform a constrained log-quadratic regression on mortality rates for the old, which they then extrapolate to the very old. Kannisto (1992) and Thatcher et al. (1998) suggest using the logistic function given that it has convenient asymptotic properties (decelerating increase in mortality rates) when it comes to model mortality rates at advanced ages.

In recent years several papers have been published using extreme value theory (EVT) to model human mortality at extremely high ages as an attractive solution for the problems of inaccuracy and unavailability of mortality data at very old ages. Extreme value theory provides a framework to formalize the study of behaviour in the tails of a distribution. Aarssen and De Haan (1994) estimated a finite upper bound on the distribution of human life spans, while Galambos and Macri (2000) argued that such an upper bound could not exist. Thatcher (1999) modeled the highest attainable age by using classical extreme value theory. Watts et al. (2006) modeled the highest attained age by using the Generalized Extreme Value (GEV) distribution. Beelders and Colarossi (2004) use EVT to model mortality risk and apply the results to the pricing of the Swiss Re mortality bond issued in 2003. Han (2005) uses EVT to model the mortality rate for the elderly. Chen and Cummins (2010) employ extreme value theory to model rare longevity events in the context of longevity risk securitization. Li et al. (2008, 2011) and Bravo and Corte-Real (2012) develop a method called the threshold life table, which integrates EVT to the parametric modeling of mortality and test it using U.S., Canadian, Japanese, Australian and New Zeland mortality data observed in a specific time period.

In this paper, we use EVT to model human mortality at extremely high ages using a unique dataset of the exact ages at death (in days), sex and birth cohort of all Portuguese residents who died between 1980 and 2018 at high age a minimum age of 90 years. Contrary to previous studies that used annual aggregated mortality data, we work with reliable mortality data from official deaths registration provided by Statistics Portugal. We restrict our analysis to extinct cohorts to ensure we deal with complete data only. To determine the threshold age, i.e., the youngest age beyond which the Generalised Pareto (GP) distribution offers a reasonable approximation of the remaining lifetime distribution, we empirically investigate alternative methods, including the threshold life table method (Li et al., 2008; Bravo et al., 2012), the Pickands (1975) method, the empirical mean excess function plot method, the Reiss and Thomas (1997) automatic selection procedure and an Extreme Value Mixture Model (Scarrott et al., 2012).

The application of EVT entails the choice of an adequate cut-off between the central part of the distribution and the upper tail, i.e., a point separating ordinary from extreme realizations of the random variable. When working with threshold exceedances, the cut-off is induced by the threshold age. This is a very delicate issue concerning statistical methods of EVT, since the choice of the threshold age entails a trade-off between the bias and the variance of parameter estimates (Embrechts et al., 2008). Contrary to previous papers, we investigate the choice of the appropriate threshold in the peaks-over-threshold (POT) method using alternative statistical methods, we conduct both a period and cohort analysis to investigate to what extent the theoretical arguments supporting the use of EVT techniques are matched in this dataset. We then use the model to statistically estimate the life table highest attained age over a period of almost forty consecutive years and analyse gender differences observed in the data. This will allow us not only to analyse the dynamics of extreme longevity risk by age and gender, but also over time. Additionally, we contribute to existing longevity risk literature by using standard time series methods to model the dynamics of the limiting age over time and to derive point forecasts and prediction intervals for the highest attained age. This is crucial in forecasting the age structure of population over time. We the used the model result to the estimate the ultimate age, the maximum age at death and the price of life annuity contracts and corresponding risk measures (VaR, Expected Shortfall).

References

Aarssen K., de Haan L., 1994. On the maximal lifespan of humans. Mathematical Population Studies 4, 259-281

Ayuso, M., Bravo, J. M., & Holzmann, R. (2017a). On the Heterogeneity in Longevity among Socioeconomic Groups: Scope, Trends, and Implications for Earnings-Related Pension Schemes. Global Journal of Human Social Sciences - Economics, 17(1), 31-57.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2017b). Addressing Longevity Heterogeneity in Pension Scheme Design. Journal of Finance and Economics. 6(1), 1-21.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2019). Getting Life Expectancy Estimates Right for Pension Policy Period versus Cohort Approach. Submitted to Journal of Pension Economics and Finance.

Beelders, O., Colarossi, D., 2004. Modeling mortality risk with extreme value theory: The case of swiss Re's mortality-indexed bond. Global Association of Risk Professionals 4, 26-30.

Boleslawski, L., Tabeau, E., 2001. Comparing theoretical age patterns of mortality beyond the age of 80. In E. Tabeau et al. (Eds). Forecasting Mortality in Developed Countries: insights from a statistical, demographical and epidemiological perspective. Kluwer Academic Publishers, pp. 127-155.

Bravo, J. M. & Real, P. C. (2012). Modeling Longevity Risk using Extreme Value Theory: An Empirical Investigation using Portuguese and Spanish Population Data. In: Proceedings of the 7th Finance Conference of the Portuguese Finance Network, July 5-7, 2012, Aveiro, Portugal. ISBN: 978-972-789-362-1. Available at https://dspace.uevora.pt/rdpc/handle/ 10174/7344

Bravo, J. M., Magalhães, M. G., Coelho, E., 2007. Mortality and Longevity Projections for the Oldest-Old in Portugal. in Working Session on Demographic Projections, Bucharest, Romania, 10-12 October, 2007, EUROSTAT (Series: Population and social conditions, Collection: Methodologies and working papers), pp. 117-132.

Buettner, T., 2002. Approaches and experiences in projecting mortality patterns for the oldest-old. North American Actuarial Journal, 6(3), 14-25.

Chen, H., Cummins, D., 2010. Longevity bond premiums: The extreme value approach and risk cubic pricing. Insurance: Mathematics and Economics 46, 150-161.

Coale, A., Guo, G., 1989. Revised regional model life tables at very low levels of mortality. Population Index 55, 613-43.

Coale, A., Kisker, E., 1990. Defects in Data on Old-Age Mortality in the United States: New Procedures for Calculating Mortality Schedules and Life Tables at the Highest Ages. Asian and Pacific Population Forum 4, 1-31.

Denuit, M., Goderniaux, A., 2005. Closing and projecting lifetables using log-linear models. Bulletin de l'Association Suisse des Actuaries, 1, 29-49.

Embrechts, P., Klüppelberg, C., Mikosch, T., 2008. Modeling Extremal Events: For Insurance and Finance. Springer-Verlag, Berlin.

Galambos, J., Macri, N., 2000. The Life Length of Humans Does Not Have a Limit. Journal of Applied Statistical Science 9(4): 253-64.

Gompertz, B., 1825. On the Nature of the Function Expressive of the Law of Human Mortality and on a New Mode of Determining Life Contingencies. Royal Society of London, Philosophical Transactions, Series A 115, 513-85.

Han, Z., 2005. Living to 100 and Beyond: An Extreme Value Study. Living to 100 and Beyond Symposium Monograph. Available at: http://www.soa.org/ccm/content/research-publications/library-publications/monographs/life-monographs/living-to-100-and-beyond-monograph/

Heligman, L., Pollard, J., 1980. The Age Pattern of Mortality. Journal of the Institute of Actuaries 107, 437-55.

Himes, C. L., Preston, S. H., Condran, G. A., 1994. A Relational Model of Mortality at Older Ages in Low Mortality Countries. Population Studies 48, 269-91.

Kannistö, V., 1992. Development of oldest-old mortality, 1950-1990: Evidence from 28 developed countries. Odense University Press.

Li, S.H., Hardy, M.R., Tan, K.S., 2008. Threshold life table and their applications. North American Actuarial Journal 12 (2), 99-115.

Li, S.H., Ng, A., Chan, W, 2011. Modeling old-age mortality risk for the populations of Australia and New Zealand: An extreme value approach. Mathematics and Computers in Simulation 81, 1325-1333

Pickands, J. 1975. Statistical Inference Using Extreme Order Statistics. Annals of Statistics 3: 119-131.

Pitacco, E., 2004. Survival models in a dynamic context: a survey. Insurance: Mathematics and Economics 35, 279-298.

Robine, J. M., Vaupel, J. W., 2001. Supercentenarians: Slower ageing individuals or senile elderly?. Experimental Gerontology 36, 915-930.

Scarrott, C., and A. MacDonald. 2012. A Review of Extreme Value Threshold Estimation and Uncertainty Quantification. REVSTAT-Statistical Journal 10: 33-60.

Thatcher, A. R., 1999. The long-term pattern of adult mortality and the highest attained age. Journal of the Royal Statistical Society 162 (1), 5-43.

Thatcher, A., Kannisto, V., Vaupel, J., 1998. The force of mortality at ages 80 to 120. Odense Monographs on Population Aging, Odense University Press, Odense.

Vaupel, J. M., 2010. Biodemography of human ageing. Nature 464, 536-542.

Watts, K. A., Dupuis, D. J., Jones B. L., 2006. An Extreme Value Analysis of Advanced Age Mortality Data, North American Actuarial Journal 10 (4), 162-177.