From Life Expectancy at Birth to Life Table. A Novel Approach

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1 Abstract

The shape of Life expectancy has always caught the researchers'interest. From the beginning of the new millennium, its evolution has led scholars to give more emphasis to life expectancy investigation. Following this line of research, we propose a novel model, based on Deep Neural Network (DNN) in order to provide unobserved mortality rates (or more general vital rates) using demographic summary measure. The present analysis has been carried out by using life expectancy at age 0 and 65 as input for years Y_i. Thus, as output, we obtain a mortality surface for the same years.

2 Introduction

The rise in human longevity during the 20th century leads a growing interest in modeling and projection of mortality rates and life expectancy alike for demographers and actuaries. The hypothesis of an imminent boundary of human life has been repeatedly disproved by empirical evidence (HMD 2019 [1], Oeppen and Vaupel [5]). In their influential work, Oeppen and Vaupel coin the "best-practice life expectancy" (BPLE) hypothesis, that is the maximum female life expectancy observed in a given calendar year which shows a linear increase at a constant pace over time since 1840. The BPLE concept leads to an increasing appeal on methods based on extrapolating life expectancy, offering a higher level of forecast accuracy with the advantages of being more easily understood, portaryng the analisys, using just a univariate time serie (eg life expectancy at birth). Many promising studies have been carried out, starting from Lee (2006)[2], Torri and Vaupel (2012) [8], Raftery et al. (2013)[7], Pascarius et al. (2018)[6], Nigri et al.

2019 (working paper)[4].

The prediction of future mortality levels by direct forecasting of life expectancy or more general, using the demographic summary measures, is much more intuitive, compared to models based on extrapolation of age-specific rates. Unfortunately, the reconstruction of vital rates from summary measures does not seem to be effortless, jeopardizing in many cases the accuracy of the estimations.

The literature shows efforts in this sense, such as the Log-quadratic model Wilmoth et al., (2012)[10] the model introduced by Ševcíková et al. (2016)[11] by adopting an inverse approach to death rates estimation starting from life expectancy. Unfortunately, none of the proposed models would seem to excel over others.

Our paper contributes to literature proposing a novel model to obtain vital rates from the demographic summary measure. We will deal with practical application in mortality field using life expectancy values, nevertheless, the method is useful in different situation such as: future target life expectancy, life tables for countries with deficient data and historical life table construction, age-specific fertility dynamics from total fertility or mean age at childbearing.

3 Data

We use data from Human Mortality Data Base[1]. All life table for all country has been selected, the model has been performed on Italy, USA, Australia, and Japan female population. The data have been arranged in a two different matrix X and Y respectively with the format year x age, in which the columns represent the value for each age and rows refer at a different year. The X matrix is composed of input data, in our case study, they refer to life expectancy at age 0 and age 65 for each year. The matrix Y contains the desiderate output that is the matrix of the mortality rates for each age and year. In order to obtain a Train - Test split, we carry out a random sampling on the matrix rows (years), in this way, we avoid the time dependence by year. Thus we use the 80% of the database for training and 20% of the database for testing our model.

4 Model

The term neural network (NN) originated as a mathematical model that replicates the biological neural networks of the human brain. NN architecture includes neurons, synaptic connections that link the neurons, and learning algorithms. Typically, NN is formed by three types of layers, respectively, called input, hidden and output layer and each one has several neurons. Each unit in a network gets "weighted" information through synaptic links from the other connected ones and returns an output by using an activation function transforming the weighted sum of input signals (for more details [3]).

In our case the model catches the input data, then it learns the hidden

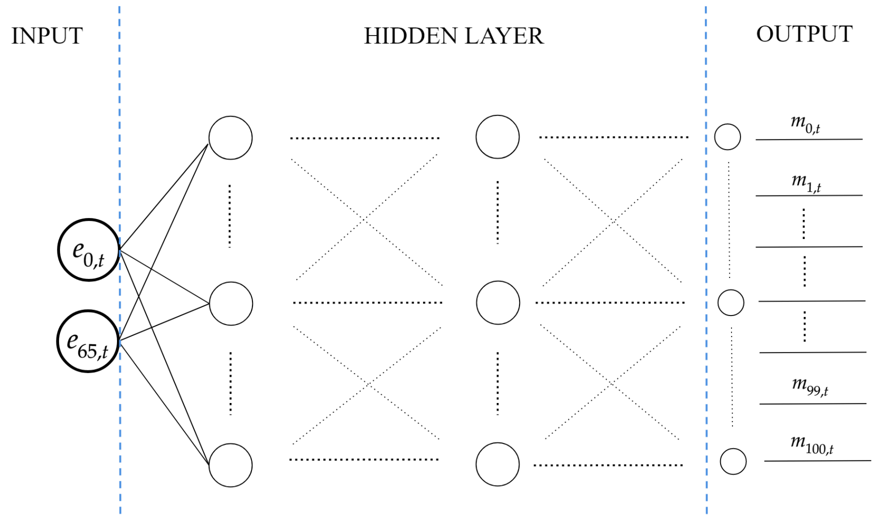


Figure 1: Graphical Model Representation.

pattern into the data, and give back the output. In doing so the model needs the life expectancy at age 0 and 65 for each year, as input data. It provides us the mortality rate (or d_x) for the same years. We perform a training-test that in this context it is slightly different from canonical backtesting exercise, in a way that, the test vector has been selected without using a time-dependent selection. Thus for sake of clarity, in the training vector, we can obtain the following years: for instance1980, 1995, 1970, 2000. Conversely, in the test vector, we can obtain the following years: 2010,1985,1999, 1997, 1975, 2001. Thus the training-test vectors do not show a time-dependent pattern.

In order to select the best combination of hyperparameters for the network, a preliminary round of fine-tuning is carried out. The best combinations, obtained in this step, are used for the DNN calibration in the forecasting procedure. After the tuning procedure, we used the architectures with six hidden layers. In each layer, we use five hundred neurons, furthermore in two layers we employed a drop out regularization with a rate of 1%. The Rectified Linear Unit (ReLU) activation function outperformed the other functions tested for all countries. During the fitting procedure, we perform two hundred epochs with a batch size equal to one. In addition, no clear evidence emerged for the influence of other hyperparameters on the performance.

The figure 1 shows a clear representation of how the model works. It accepts the values of life expectancy at birth and at age 65 as input. Then it provides the mortality surface. Furthermore, the model could provide any other desired demographic measure, such as the age at death distribution.

5 Result

The analysis includes numerical and graphical processing of the goodness of fit. In particular, we follow the out of sample approach that represents the testing step in the machine learning field.

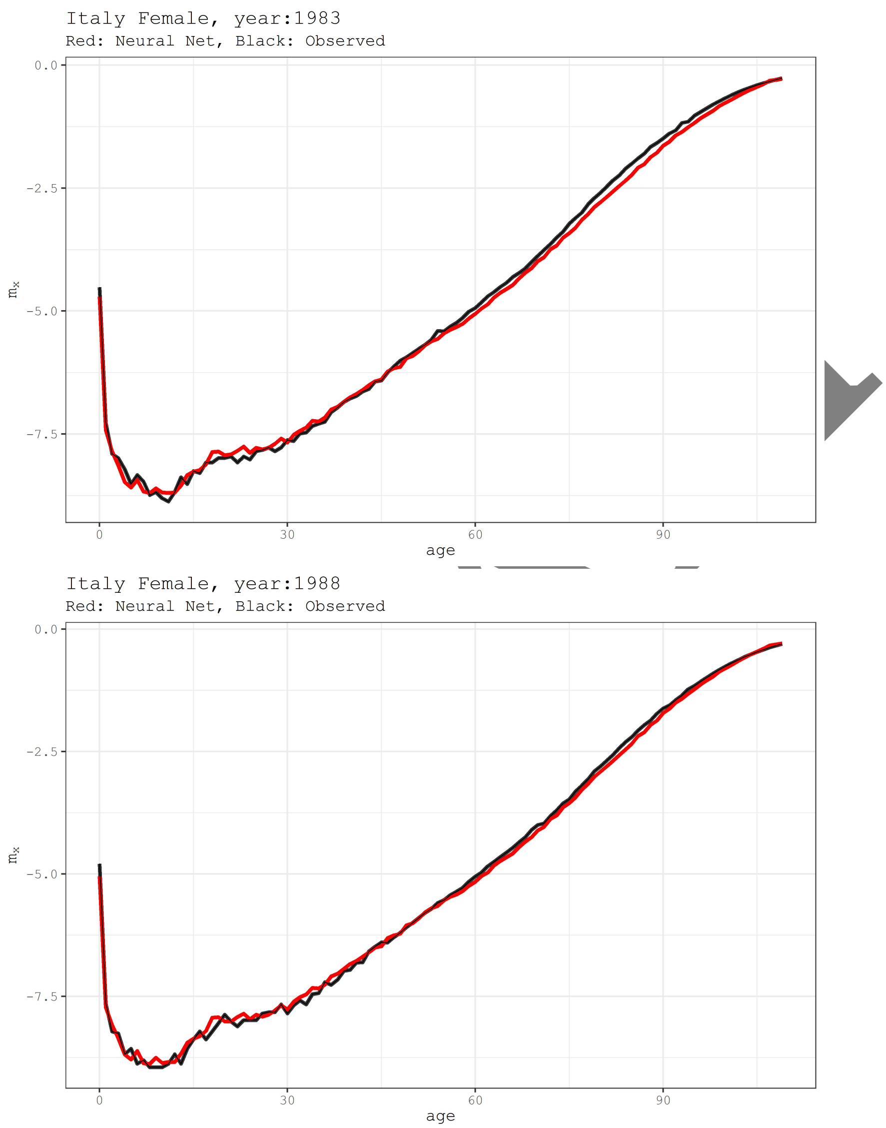
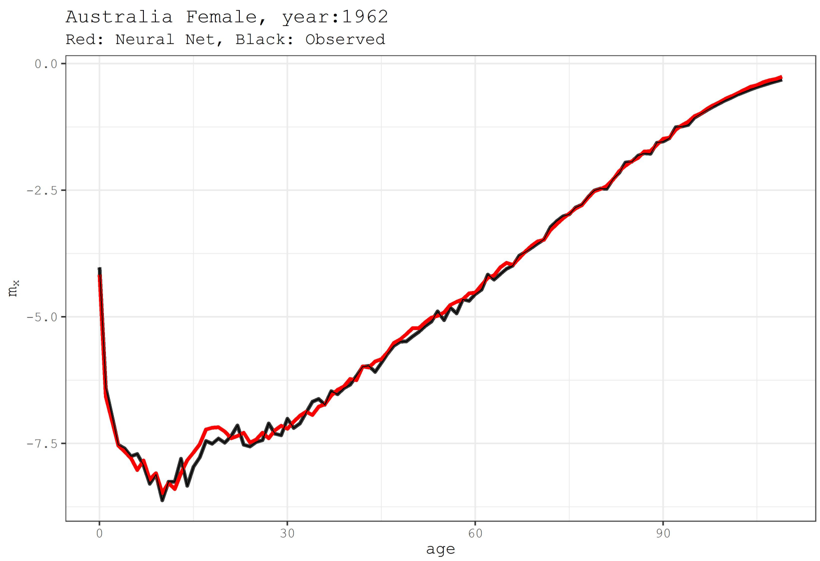


Figure 2: ITALY m_x.



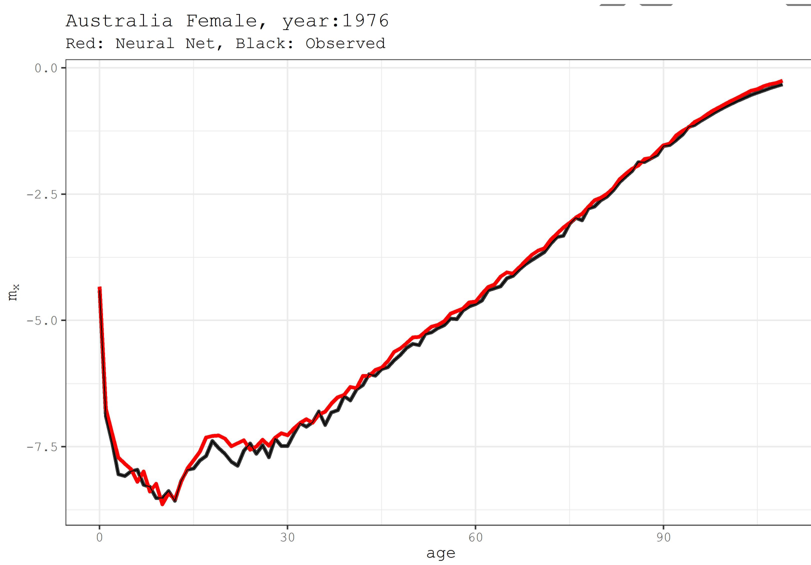


Figure 3: AUSTRALIA: m_x.

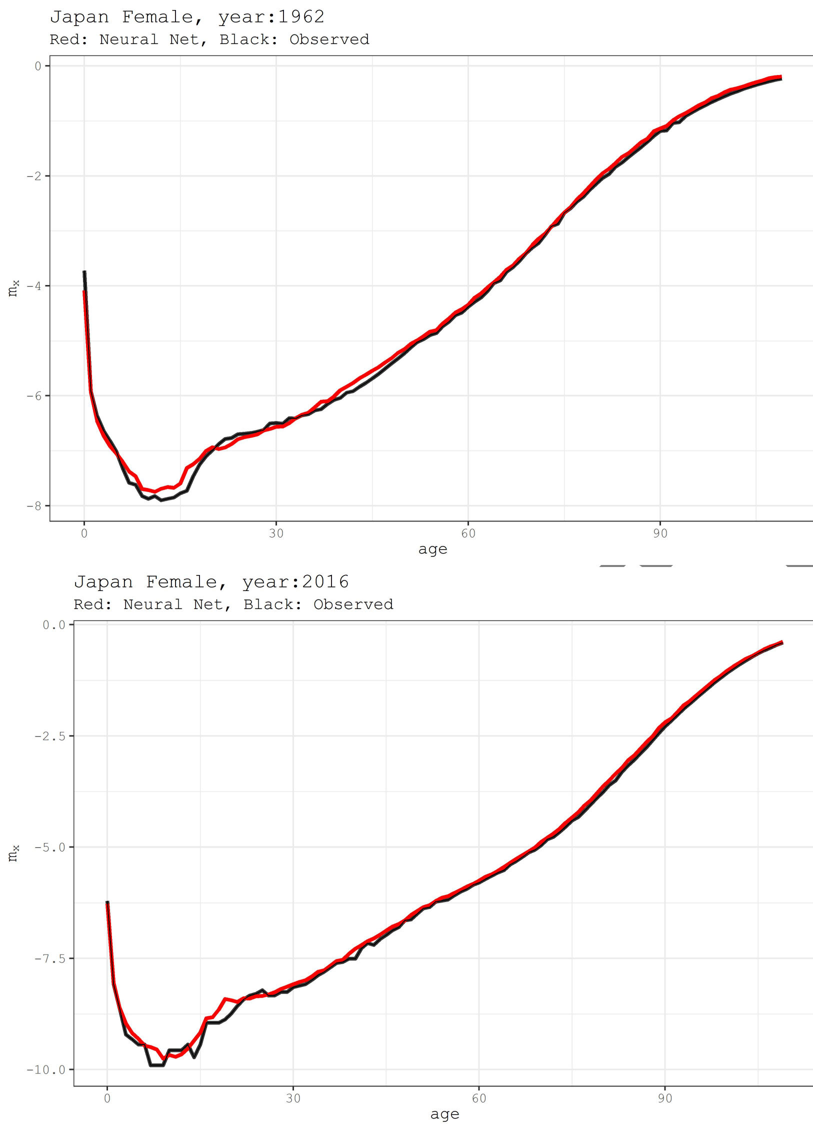


Figure 4: JAPAN: m_x.

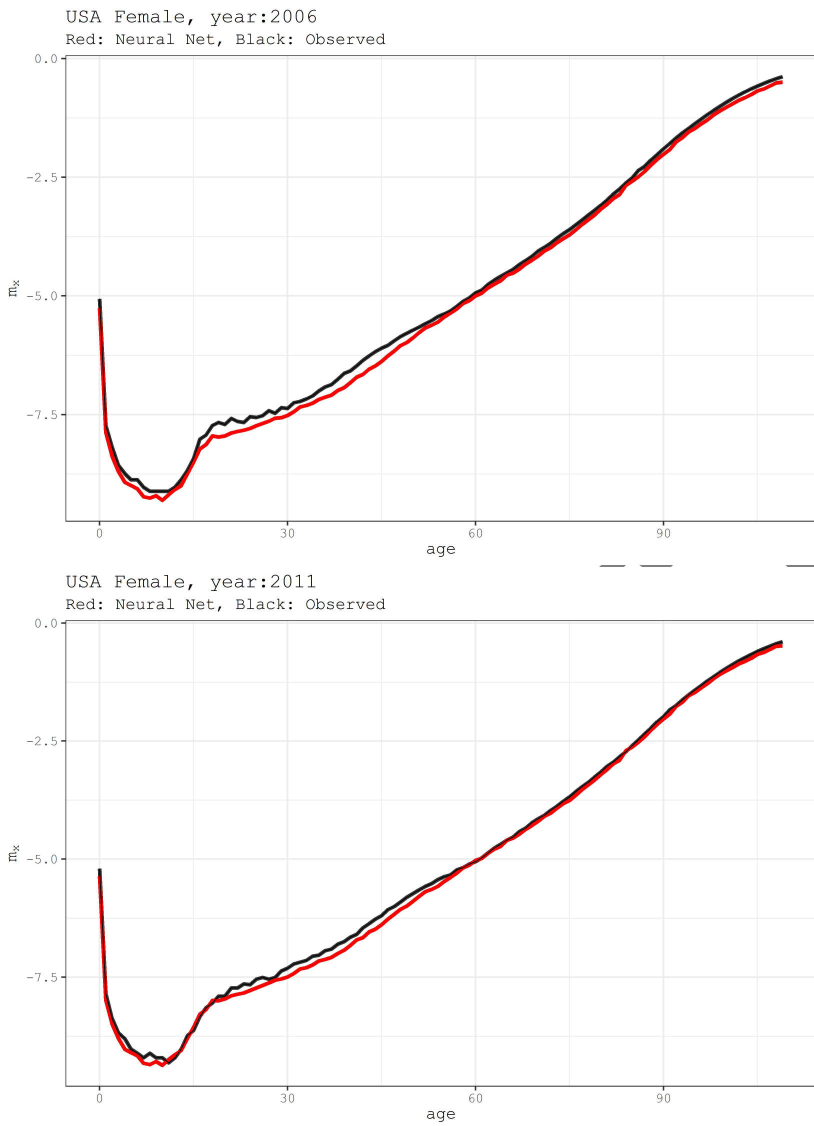


Figure 5: USA: m_x.

Once the motility surface is obtained, we can use smoothing in order to manage the data noise level. In the figure 6 below we see an example of a smoothed m_x estimation. In the same figure, we also used our model to get the age at death distribution

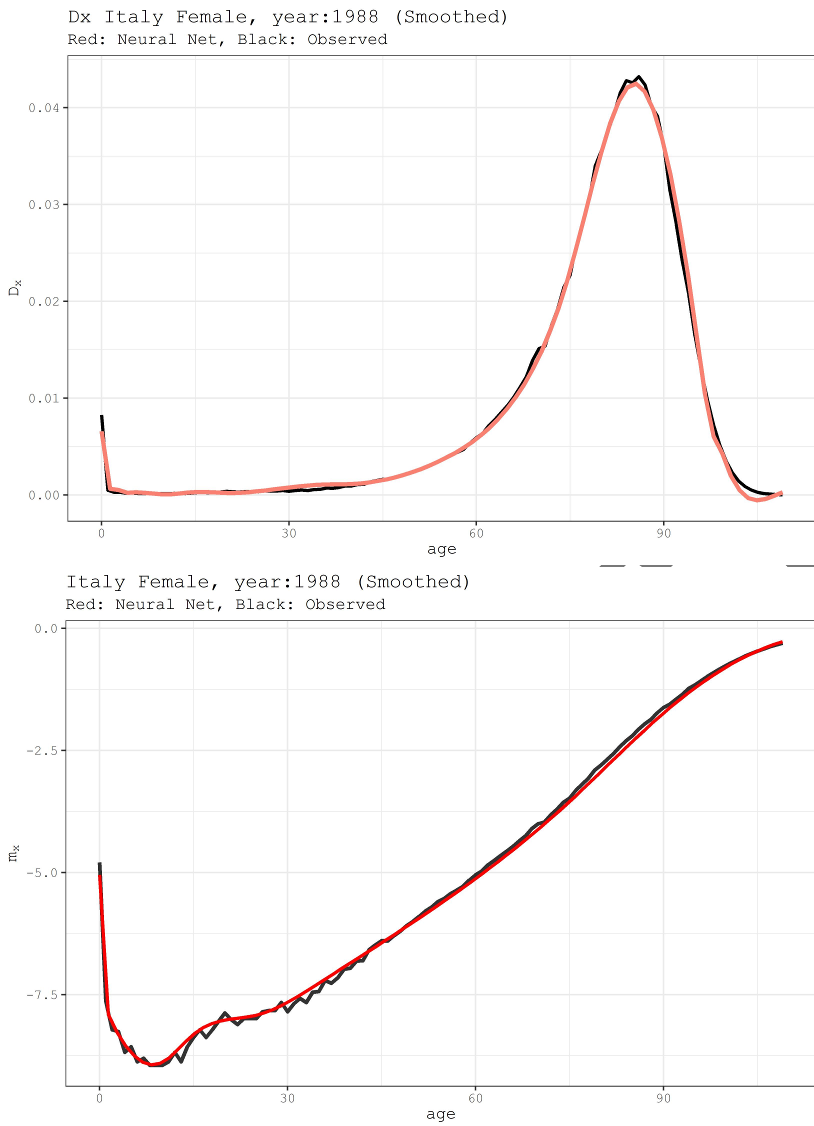


Figure 6: ITALY 1988: d_x and m_x.

Besides graphical check, we calculate the following goodness of fit measures:

$$\textbf{Mean Absolute Error (MAE):} \sum^{n} \frac{\mid m_{x,t} - \hat{m_{x,t}} \mid}{n}, \tag{1}$$

$$\textbf{Root Mean Square Error (RMSE):} \quad \sqrt{\frac{\sum^{n} (m_{x,t} - \hat{m_{x,t}})^2}{n}}. \tag{2}$$

Table 1: Performance of DNN the testing set for each country.

Country	Female	
Australia	*MAE*	*RMSE*
$m_{x,t}$	0.175	0.181
Italy	*MAE*	*RMSE*
$m_{x,t}$	0.141	0.152
Japan	*MAE*	*RMSE*
$m_{x,t}$	0.126	0.132
USA	*MAE*	*RMSE*
$m_{x,t}$	0.123	0.131

In the mortality field, theories that embrace the steady increase in life expectancy has been extensively presented. In light of that, the latter point should play an important role in the forecasting approach. For this reason, the estimation of the vital rates from a nonlinear forecasted summary demographic measure, should represent a crucial point. Furthermore, in developed countries, mortality deceleration is lead by a decline at younger ages as well as an accelerating at old ages. This phenomenon leads to a remarkable drawback among the extrapolative models since the

fixed structure of the

β_x index over time (Lee and Miller, Girosi and King, Li et al.). Our approach will be able to overcome this problem assuming the future values of mortality surface are a function (estimated by DNN) of nonlinear and historically coherent forecasting (Nigri et al. 19 [4]) of life expectancy at birth and at 65. The investigation has been performed on four countries throughout the world and by gender. The proposed approach shows very high accuracy performance thanks to deep architecture, by which we can investigate the hidden relationship in our data. These features allow the DNN to provide more accurate forecasting, coherent with observed data.

6 Conclusion

In this draft, we provide a novel view in order to forecast mortality (or vital) rates. Using a combination of Deep Neural network and B-spline smoothing we are able to catch the hidden pattern and the nonlinear relationship between the summary demographic measures and age period rates. We can consider an important application in the context of incomplete data such as fertility history or migration. We also consider further development combining the use of life expectancy (and lifespan inequality) forecasting model by Nigri et. al 2019 [4] in order to obtain unobserved data more coherent whit past trend. Thus we obtain the future mortality surface from the e_0 and e_65 forecasted value.

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