# The relationship between longevity and lifespan variation

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## 1 Introduction

In the 20th century, developed countries have been experiencing a remarkable decline in mortality at all ages driven by medical progress but also by greater spread of prevention, healthy lifestyles and, in general, by better living conditions. After the 2nd World War death rates start to significantly decrease at adults and older ages, thanks above all to scientific advancements against chronic diseases (Crimmins et al., 2010). Since then, mortality has never stopped decreasing even at increasingly higher ages (Rau et al., 2008; Vaupel, 1997). As a result, life expectancy at birth has considerably increased over time and, for most developed countries, it even doubled during the last century (HMD, www.mortality.org). As there is no evidence of a current deceleration in mortality decline, life expectancy's growth is expected to continue in the next future, giving scope to the most optimistic views about the maximum human life expectancy.

During the past two centuries or so, the causes of death against which progress has been made have shifted from infectious diseases to chronic diseases and, consequently, the ages at which mortality has been reduced have shifted from childhood to old ages. For developed countries, recent gains in life expectancy have been largely determined by reductions in elderly mortality. Furthermore, the transition from high to low mortality has been accompanied by rapid and intense compression, or in other words by an increase of lifespan equality, in which the increase in the modal age-at-death corresponds to the decrease in the dispersion of the age-at-death distribution around its modal value (Kannisto, 2000).

Several studies have addressed the question of lifespan equality and its relationship with life expectancy providing somehow mixed results. Aburto and Van Raalte (2018) have explored the lifespan variation in the Central and Eastern Europe, an interesting case study because of its considerable instability in mortality patterns, showing that life expectancy and life disparity varied independently from each other, largely because of different mortality trends depending on age. On the contrary, Vaupel et al. (2011), by analysing the life table of 40 developed countries, found a strong negative correlation between high life expectancy and low lifespan variation.

Many aspects of the positive evolution experienced in recent decades need further investigation. The reduction of mortality at increasingly older ages could mean for some low mortality country, as well as for other low-mortality countries, the entry into a phase that sees stagnation in the process of mortality compression at old ages where the age-at-death distribution shifts to increasingly higher ages but is shape remains unchanged over time.

Furthermore, as already observed for Japan (Cheung and Robine, 2007),

Our paper contributes to the literature by further investigating the underlying mechanisms that have governed the secular mortality improvement. We study the shape of the whole life expectancy distribution including the tails, by using expetile regression, and analyse it taking into account mortality compression. This might reveal latent dynamics that could not emerge by merely focusing on the central trend of the distribution.

## 2 Expectile distribution of life expectancy and variation of age of death

In order to investigate the relationship between life expectancy and lifespan variation, we examined the estimated expectiles of the entire distribution of the life expectancy based on all countries included in the HMD together with selected measures of mortality compression.

While life expectancy has been proved to hide heterogeneity in individual mortality paths, a lifespan variation indicator measures heterogeneity at the population level and uncertainty in the age-at-death at the individual level. In other words, a lifespan variation indicator quantifies the inequality within a population and the uncertainty at age-at-death at individual level (Aburto and Van Raalte, 2018). According to several studies (Wilmoth and Horiuchi, 1999; Edwards and Tuljapurkar, 2005; Colchero et al., 2016; Nemeth, 2017), the lifespan variation measures appear to be negatively correlated to life expectancy at birth. When life expectancy increases, lifespan variation tends to decrease. However, the strength and the form of this relation significantly varies across populations and over time.

A number of inequality measures of mortality compression have been proposed in literature to analyse lifespan variation (Wilmoth and Horiuchi, 1999; Kannisto, 2000; Shkolnikov et al., 2003; Thatcher et al., 2010; van Raalte and Caswell, 2013). Here we consider Keyfitz's measure of entropy in the life table, which expresses the degree of concavity in the survivorship curve. Keyfitz's entropy, commonly denoted with $H$, measures the relative variation in the length of life with respect to life expectancy at birth, i.e. the life expectancy elasticity to a change in mortality:

$$H(t) = -\frac{\int_0^\infty S(x,t)\ln S(x,t)dx}{\int_0^\infty S(x,t)dx} \tag{1}$$

where $S(x,t)$ is the survival function at age $x$ and time $t$.

Keyfitz's entropy can be also written as a function of life expectancy, $e_0(t)$, and life disparity that is the life expectancy lost due to death by an individual aged x at time t. As defined by Vaupel et al. (2011), lifespan variation measures how much lifespans differ among individuals or, in other words, the dispersion in the age-at-death. For instance, when mortality is highly variable, some individuals will die at a much lower age than the expected age-at-death, contributing many lost years to life disparities. Conversely, when mortality is highly concentrated at older ages, life disparity decreases.

Denoting by $e_0^\dagger(t) = -\int_0^\infty S(x,t)\ln S(x,t)dx$ the life disparity and $e_0(t)$ the life expectancy at birth, is it possible to provide a very useful alternative formulation of the Keyfitz's entropy:

$$H(t) = \frac{e_0^\dagger(t)}{e_0(t)} \tag{2}$$

The minimum value of $H(t)$ is an indicator of the minimum heterogeneity among individuals in a population, corresponding to the maximum compression of mortality. Therefore, the Keyfitz's entropy can be considered an indicator of mortality compression as well.

As recent gains in life expectancy are mainly driven by mortality reduction at old ages, it is worth to examine the variability of deaths above the modal age, i.e. the old-age compression, by analyzing the standard deviation of the age distribution of deaths above the mode, $SD(M+)$ (Cheung and Robine, 2007; Canudas Romo, 2010; Thatcher et al., 2010):

$$SD(M+) = k \cdot e(M) \tag{3}$$

where $k$ is the ratio of $SD(M+)$ to $e(M)$, which can be assumed constant and equal to 1,233 as proved by Thatcher et al. (2010), and $M$ is the modal age at death:

$$M = X + \frac{d(X) - d(X-1)}{[d(X) - d(X-1)] + [d(X) - d(X+1)]}, \tag{4}$$

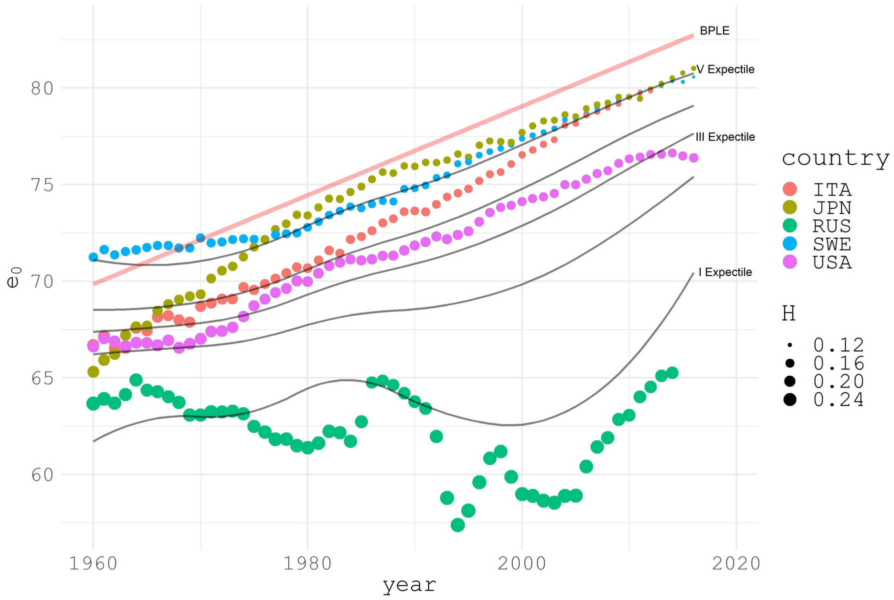
$X$ is the age which has the highest number of deaths and $e(M)$ is the mean of the model age at death:

$$e(M) = e(X) * (X + 1 - M) + e(X + 1) * (M - X). \tag{5}$$

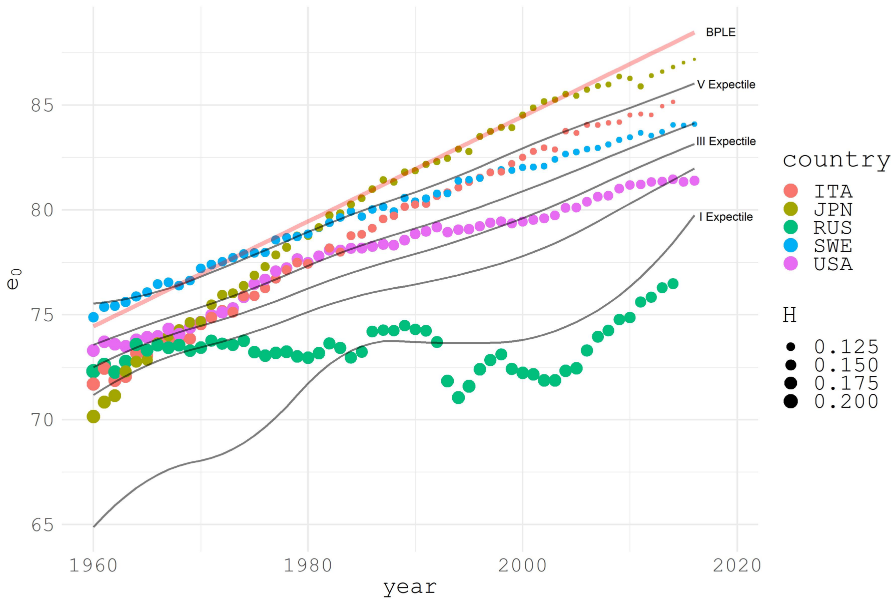
Fig 1 shows the expectile curves (black lines) of the entire distribution of the life expectancy, Oeppen and Vaupel Oeppen and Vaupel (2002) estimated line of BPLE (red line), and the observed values of the life expectancy for Italy, Japan, Russia, Sweden and U.S.A (coloured dots), from 1960 to 2016, separately for women and men. The size of the dots returns the degree of the compression of the age at death distribution of a country: the smaller the dot the greater the compression of the considered country. This graph links the evolution of life expectancy of specific countries, and their placement over time with respect to the BPLE, with the long term process of compression that, with different speed and intensity depending on the country, has accompanied the demographic transition towards high level of life expectancy. Panels (a) and (b) represent the relation between life expectancy at birth and Keyfitz's entropy, respectively for men and women while panels (c) and (d) show the relation of life expectancy with the standard deviation above the mode, for male and female respectively.

As shown in the picture, Japan shifts very rapidly from the bottom (second expectile) to the higher tail of the distribution (fifth expectile) but already from 1980 the increase of life expectancy decelerates and at the beginning of the twenty-first century it stagnates especially for women for whom life expectancy diverge from BPLE's line without however losing its leading position. This process has been accompanied by overall mortality compression, i.e. a decreasing trend of the H index. However, when looking at old-age mortality compression, i.e. the standard deviation above the mode, the picture shows a less intense and rapid process for both sexes that at the end of the twenty century almost comes to an halt, confirming thus the results of Cheung and Robine (2007). Furthermore, for recent years, women in particular experience higher old-age compression in correspondence of stagnation in life expectancy that seems however slightly increasing as life expectancy starts to grow again. Russia, located on the lower tail of the distribution (first expectile), shows an oscillatory trend in life expectancy for both women and men, which is associated with a lower compressed age-at-death distribution and since 1980 a quite constant trend. Old-age mortality compression varies more but without a precise trend. This behaviour shows on one hand the high heterogeneity of the Russian population, on the other hand the independence of the life expectancy at birth from the life disparity in the eastern countries, confirming thus the findings of Aburto and Van Raalte (2018). US show an increase of life expectancy interspersed by periods of stagnation that, after an initial gain, make it lose some positions, settling down in the middle of the life expectancy distribution (third expectile). As in the case of Russia, mortality compression is quite stable for both sexes. However, particularly for men, old-age mortality compression is more variable and shows a modest tendency to decline at least until the stagnation of life expectancy observed in the last decade or so. Italy and Sweden are currently both very close to the upper tail of the distribution, especially when considering male life expectancy (fifth expectile). However while in Sweden life expectancy has maintained its position around the extreme tail over the long period, although female life expectancy slightly shifts downward in recent years, Italian life expectancy, for both women and men, shifts considerably upward in the distribution during the study period (from second to fifth expectile). This very positive trend has been accompanied by massive and rapid compression, more rapid than those found for Sweden. Regarding the standard deviation above the mode, again there is not a regular pattern. However, for recent years, old age at death distribution does not appear to be significantly more compressed, especially for women.

Our findings strongly support the existence of a negative relationship between longevity and lifespan variation. Furthermore, our study shows that the strength of the overall compression has been more significant for those countries that have radically changed their position in the distribution of life expectancy, that is mortality compression has been as rapid and intense as the gains in life expectancy have been more important. In the study period entropy has continued to decline because mortality improvements that have occurred below an age close to life expectancy have exceeded improvements at oldest ages (Aburto et al., 2019). In contrast, old-age distribution of deaths, considering only deaths above the modal age, has maintained more or less its shape in recent years, quite independently by the country position in the life expectancy distribution and despite a continuous increase of the modal age at death, although in some cases its rate of increase has decelerated in the very last years. Thus, for low-mortality countries, the age distribution of old age mortality seems to continue to shift to older and older ages with a nearly constant standard deviation. This is in keeping with those recent studies showing that the front of longevity may continue to advance (Zuo et al., 2018; Medford, 2019).

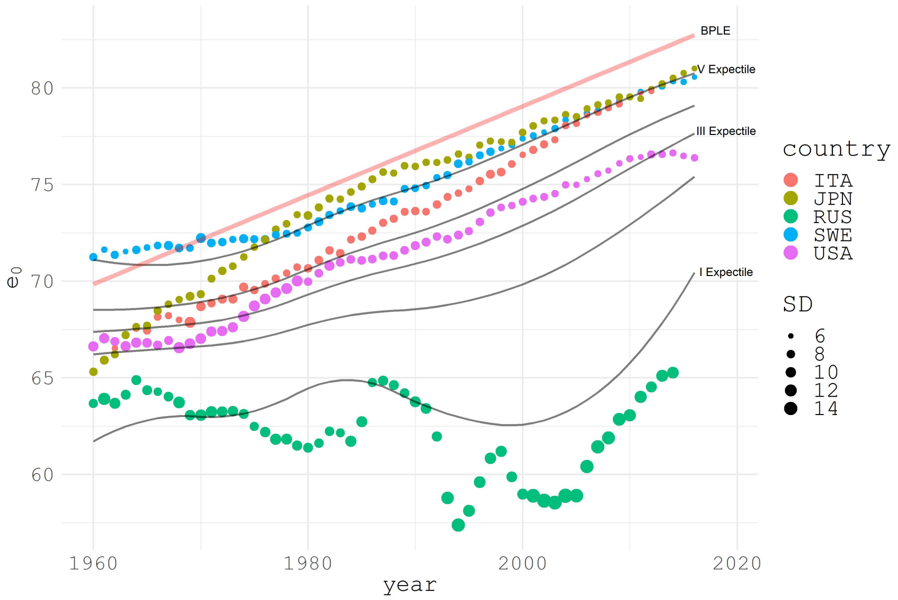



(a) Male e(x) - H

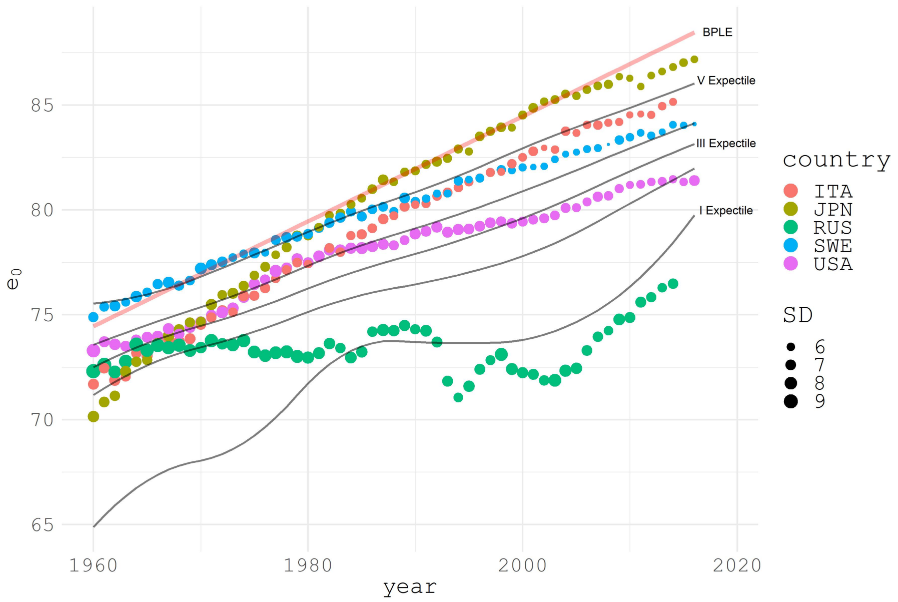



(b) Female e(x) - H

Figure 1: Smoothed expectiles of life expectancy and $H(t)$. Focus on years 1960-2016. Countries: Japan, Russia, Spain, Sweden and USA.




(a) Male e(x) - SD+




(b) Female e(x) - SD+

Figure 2: Smoothed expectiles of life expectancy and $SD(M+)$. Focus on years 1960-2016. Countries: Japan, Russia, Spain, Sweden and USA.

## 3 Conclusion

This study investigates the long-term dynamics of life expectancy in the developed world taking into account the specific contribution of each country, and how this has changed over time. We have applied a smoothed expectile regression whose main advantage is, beside that to analyse the entire distribution of the life expectancy including the tails, to provide an estimation of the means. Furthermore, we have examined the distribution of the life expectancy in light of the variability of the age-at-death distribution with a focus on old-age deaths.

Our results provide a comprehensive picture of the different phases and transitions experienced by developed countries in the evolution of the life expectancy that have led to a continuous increase in the best practice life expectancy. After a period of great heterogeneity, low-mortality countries converged to a more homogeneous behaviour in the progress against mortality. Still, the analysis of the life expectancy distribution, combined with that of mortality compression, reveals country-specific dynamics over time. Overall, we found clear evidence of a negative relationship between longevity and lifespan variation, confirming thus previous studies. The strength of this relation varies depending on the phase in which the country is located within the life expectancy distribution and how fast it converges to the best performance. However, when focusing on old-age deaths, variation of the age at death appears more stable regardless the position of the country within the life expectancy distribution. With a continuous increase of the modal age at death and a somehow stagnating process of old-age compression, one cannot say that longevity is approaching a limit. Will the modal age at death continue to shift to increasingly older ages in the future, meaning that an increasing number of individuals will reach higher and higher ages? Or rather, is the deceleration of this process, which has been detected in the last years, a sign that future gains in longevity could be possible only thanks to new records attained from exceptionally long-lived selected individuals, meaning that the right tail of the age-at-death distribution will extend to more extreme ages? How these mechanisms will be reflected on future trends of life expectancy? Analyses of long-lived individuals in addition to ecological studies are needed for a fully understanding of the mechanisms by which human longevity is ruled.

## References


Aburto, J. M., Basellini, U., Kjaergaard, S., and Vaupel J. W. (2018). Life expectancy and lifespan equality: a long-run relationship. Working paper.

Aburto, J. M., Alvarez, J.A., Villavicencio, F., Vaupel J.W. (2019). The threshold age of the lifetable entropy. Demographic research 41 - article 4,83–102

Aburto, J.M., van Raalte, A. (2018). Lifespan Dispersion in Times of Life Expectancy Fluctuation: The Case of Central and Eastern Europe. Demography 55: 2071-2096.

Alfó, M., Salvati, N., Ranalli, M.G. (2016). Finite mixtures of quantile and M-quantile regression models. Statistics and Computing.

Canudas-Romo, V. (2010). Three measures of longevity: time trends and record values.Demography. 2010 May;47(2):299-312.

Cheung, S. L. K., Robine, J.M. (2007). Increase in common longevity and the compression of mortality: The case of Japan. Population Studies, vol. 61, no. 1, 2007, pp. 85–97. JSTOR.

Colchero, F., Rau, R., Jones, O.R. et al. (2016). The emergence of longevous populations. Proceedings of the National Academy of Sciences (PNAS), 113(48): E7681-E7690.

Crimmins, E., Kim, J.K., Vasunilashorn, S. (2010). Biodemography: new approaches to understanding trends and differences in population health and mortality. Demography, 47(Suppl 1): S41.

Edwards, R. D., Tuljapurkar, S. (2005). Inequality in Life Spans and a New Perspective on Mortality Convergence Across Industrialized Countries. Population and Development Review, 31(4): 645-674.

Eilers, P.H.C. (2013). Discussion: the beauty of expectiles. Statistical Modelling, 13 (4): 317–322.

Eilers, P.H.C., Marx, B.D. (1996). Flexible smoothing with B-splines and penalties. Statistical Science.

Huber, P. J. (1972). Robust Statistics: A review. Annals of Mathematical Statistics, 43: 1041–1067.

Human Mortality Database (HMD). University of California, Berkeley (USA) and Max Planck Institute for Demographic Research (Germany). (www.mortality.org)

Kannisto V. (2000), Measuring the Compression of Mortality, Demographic Research, Vol.3, 6.

Koenker, R., Bassett, G. Jr. (1978). Regression Quantiles. Econometrica, 46 (1): 33-50.

Jones, M.C. (1994). Expectiles and M-quantiles are quantiles. Statistics & Probability Letters, 20 (2): 149–153

Liu, J., Li, J. (2019). Beyond the highest life expectancy: construction of proxy upper and lower life expectancy bounds. Journal of Population Research. Published on line: 27 march 2019.

Medford, A., Christensen, K., Skytthe, A., Vaupel, J.W. (2019). A Cohort Comparison of Lifespan After Age 100 in Denmark and Sweden: Are Only the Oldest Getting Older? Demography, 56: 665–677.

Nemeth, L. (2017) Life expectancy versus lifespan inequality: a smudge or a clear relationship? PLoS ONE 12 (9): e0185702.

Newey, W., Powell, J. (1987). Asymmetric Least Squares Estimation and Testing. Econometrica, 55(4): 819–847.

Oeppen, J., Vaupel, J. W., (2002). Broken limits to life expectancy. Science, 296 (5570): 1029–1031.

Pewsey, A. (2000). Problems of inference for Azzalini's skew-normal distribution. Journal of Applied Statistics, 27(7): 859–870.

Preston, S.H., Heuveline, P., Guillot, M. (2000). Demography: Measuring and Modeling Population Processes. Blackwell Publishing.

Rau, R., Soroko, E., Jasilionis, D., Vaupel, J. W. (2008). Continued Reductions in Mortality at Advanced Ages. Population and Development Review, 34: 747-768

Schnabel, S.K. and Eilers, P.H.C. (2009a). Optimal Expectile Smoothing. Computational Statistics and Data Analysis, 53: 4168–4177.

Schnabel, S.K. and Eilers, P.H.C. (2009b). An analysis of life expectancy and economic production using expectile frontier zones Demographic research, 21 (article 5): 109–134.

Schnabel, S.K. and Eilers, P.H.C. (2013). A location-scale model for noncrossing expectile curves. Stat, 2: 171–183.

Shkolnikov, V.M., Andreev, E.M., Begun, A.Z. (2003). Gini coefficient as a life table function: Computation from discrete data, decomposition of differences and empirical examples. Demographic Research, 8 (article 11): 305–358.

Shkolnikov, V.M., Andreev, E. M., Zhang, Z., Oeppen, J., Vaupel, J.W. (2011). Losses of expected lifetime in the United States and other developed countries: Methods and empirical analyses. Demography, 48: 211–239.

Thatcher, R., Cheung, K., Horiuchi S., Robine, J.M. The compression of deaths above the mode. Demographic research, 22 (article 17): 505–538.

Vallin, J., Meslé, F. (2009). The Segmented Trend Line of Highest Life Expectancies. Population and development review, 35 (1): 159–187.

van Raalte, A.A., Caswell, H. (2013). Perturbation analysis of indices of lifespan variability. Demography, 50: 1615–1640.

Vaupel J. W. (1997). The remarkable improvements in survival at older ages. Philosophical Transactions of the Royal Society of London, Series B: Biological Sciences, 352: 1799–1804.

Vaupel, J. W., Zhang, Z., van Raalte, A. (2011). Life expectancy and disparity: an international comparison of life table data. BMJ Open 1(1).

Wilmoth, J.R., Horiuchi, S. (1999). Rectangularization Revisited: Variability of Age at Death within Human Populations. Demography, 36(4): 475–495.

Zhang, Z., Vaupel, J.W. (2009). The age separating early deaths from late deaths. Demographic Research, 20 (article 29): 721–730.

Zuo, W., Jiang, S., Guo, Z., Feldman, M.W., and Tuljapurkar, S. Advancing front of old-age human survival. PNAS October 30, 2018 115 (44) 11209-11214